Non-resonant nonlinear coupling of magnetohydrodynamic waves in inhomogeneous media

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A new mechanism for the enhanced generation of compressible fluctuations by Alfvén waves is presented. A strongly nonlinear regime of Alfvén wave phase-mixing is numerically simulated in a one-dimensionally inhomogeneous plasma of finite temperature. It is found that the inhomogeneity of the medium determines the efficiency of nonlinear excitation of magnetoacoustic waves. The level of the compressible fluctuations is found to be higher (up to the factor of two) in inhomogeneous regions. The amplitude of the generated magnetoacoustic wave can reach up to 30% of the source Alfvén wave amplitude, and this value is practically independent of the Alfvén wave amplitude and the steepness of Alfvén speed profile. The highest amplitudes of compressible disturbances are reached in plasmas with β of about 0.5. The further growth of the amplitude of compressible fluctuations is depressed by saturation.

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Magnetohydrodynamic (MHD) waves play an important role in the dynamics of laboratory, space and astrophysical plasmas (e.g. [1]). The simultaneous existence of several MHD wave modes, three, at least, in the simplest case, stimulates the interest to processes of mode coupling and interaction. In particular, the interaction of linearly incompressible Alfvén waves with compressible magnetoacoustic waves is essential in problems such as the heating of the open corona of the Sun and acceleration of the solar wind. Indeed, Alfvén waves are often named as the carriers of the energy from the lower layers of the solar atmosphere to the corona. Also, in the inner heliosphere, Alfvén waves represent the main component in MHD turbulence [2]. However, compressible waves are subject to more efficient dissipation, as they decay on volume viscosity, not shear as to incompressible Alfvén waves, and the difference of these two viscosities can be several orders of magnitude. Moreover, this mechanism allows energy to be transported across field lines in contrast to Alfvén waves which can only transport energy along the field. Also, compressible waves perturb the plasma density and consequently can be detected, again in contrast to Alfvén waves, with imaging telescopes.

A classical example of such interaction is the decay instability of Alfvén waves, connected with the *resonant* three-wave interaction of Alfvén and magnetoacoustic waves [3]. The efficiency of this phenomenon is determined by the amplitudes of the interacting waves. However, this mechanism works only for quasi-periodic (perhaps wide-spectrum, [4]) waves, and is not so prominent for wave pulses that could be generated e.g. by some explosive events such as solar flares, coronal mass ejections, etc.

In contrast, *non-resonant* mechanisms for compressible fluctuation excitation are independent of the wave spectrum and can be efficient even for short wave pulses. An example of *non-resonant* MHD wave interaction is the nonlinear excitation of longitudinal magnetoacoustic perturbations by nonlinear elliptically polarized Alfvén waves through the ponderomotive force. This phenomenon is actually the generation of the second harmonic and, in contrast with three wave resonant interaction, and does not require the daughter wave to be present in the system from the very beginning. Also, the efficiency of this process is restricted by the source wave amplitude, but does not require the source wave to be harmonic. Another parameter affecting the efficiency is β, the thermal to magnetic pressure ratio. The generation of the compressible perturbations leads to self-interaction and consequent steepening of the Alfvén wave, whose evolution is described by the Cohen-Kulsrud equation [5].

Yet another non-resonant mechanism for the generation of compressible fluctuations arises when the Alfvén speed varies across the magnetic field. Initially plane, linearly polarized Alfvén waves become oblique and sharp gradients in the direction across the field are secularly generated. This phenomenon is known as Alfvén wave phase mixing, and it has been intensively studied in the context of the solar coronal heating problem [6] and often seen in full MHD numerical simulations (e.g. [7], [8]). In the compressible regime, phase mixing of Alfvén waves is accompanied by nonlinear generation of fast magnetoacoustic waves [9]. In this phenomenon, the efficiency of *nonlinear coupling* is dramatically affected by the *inhomogeneity* of the medium. To illustrate this, consider a low-β plasma with the straight and uniform magnetic field $\mathbf{B}_0 = B_0\mathbf{e_z}$, where B_0 is the absolute value of the unperturbed field and $\mathbf{e_z}$ is the unit vector in z-direction. The plasma mass density $\rho_0(x)$ varies across the field, in the x-direction. Thus, the Alfvén speed $C_A(x) = B_0[4\pi\rho_0(x)]^{-1/2}$ also varies across the field. In this geometry, Alfvén waves (perturbing V_y and B_y) and fast magnetoacoustic waves (perturb V_x, B_x, B_z components of the bulk velocity, magnetic field and the mass density ρ) are linearly coupled if the Alfvén waves have

a component of $\vec{k}$ in the y-direction [10]. However, when $\partial/\partial y$ tends to zero, the waves become *linearly* decoupled from each other and in this case, the effect of nonlinear coupling becomes pronounced. The initial stage of fast magnetoacoustic wave generation by transverse gradients in weakly nonlinear Alfvén waves is described by the equation

$$\frac{\partial^2}{\partial t^2}B_x - C_A^2(x)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)B_x = \frac{B_0}{8\pi\rho_0(x)}\frac{\partial^2}{\partial x\partial z}B_y^2. \tag{1}$$

The right handside of Eq. (1) in the Fourier transform domain is proportional to $k_z k_x A^2$, where k_z and k_x are the components of the wave number in the longitudinal and transverse directions respectively and A is the Alfvén wave amplitude. Consequently, even in the case of homogeneous Alfvén speed ($C_A = const$), a non-plane ($k_x \neq 0$) Alfvén wave generates compressible perturbations, which, according to the left hand side of Eq. (1), propagate isotropically.

In the weakly nonlinear regime the Alfvén wave steepening can be neglected and the linear solution can be used,

$$B_y = Af[z - C_A(x)t], \tag{2}$$

where f is an arbitrary smooth function prescribed by the initial conditions.

With an inhomogeneous Alfvén speed, the Alfvén wave is subject to phase mixing and $k_x \to \infty$ as time progresses. Consequently, the phase mixing *amplifies* the right hand side term of Eq. (1) and affect MHD wave coupling. Indeed, with the Alfvén wave given by Eq. (2), the right hand side of Eq. (1) becomes

$$\mathrm{RHS}(1) \propto A^2\frac{dC_A(x)}{d\,x}k_z^2 t \tag{3}$$

and, consequently, it grows *secularly* in time, proportionally to the product of the Alfvén wave amplitude and the inverse characteristic spatial scale of the medium inhomogeneity (see [9], [11] for details). The secularity in the nonlinear generation of the fast waves is entirely connected with the inhomogeneity of medium. Thus, even if the Alfvén wave amplitude is weak, *the generation of fast wave can be dramatically amplified by the medium inhomogeneity.* In the homogeneous case, the same ponderomotive force can also generate perpendicular compressible fluctuations if the Alfvén wave is initially non-plane. However, in this case, the perpendicular gradients in the Alfvén wave remain as prescribed by the initial conditions and the generation of compressible fluctuations is limited by the initial shape of the wavefront. The induced longitudinal motions, also generated by the Alfvén wave, and do not grow secularly either.

The secular generation of fast magnetoacoustic waves by phase-mixed Alfvén waves in *initial* stage of the wave interaction was observed in full-MHD numerical simulations [9], [7]. The adequate modelling of the developed

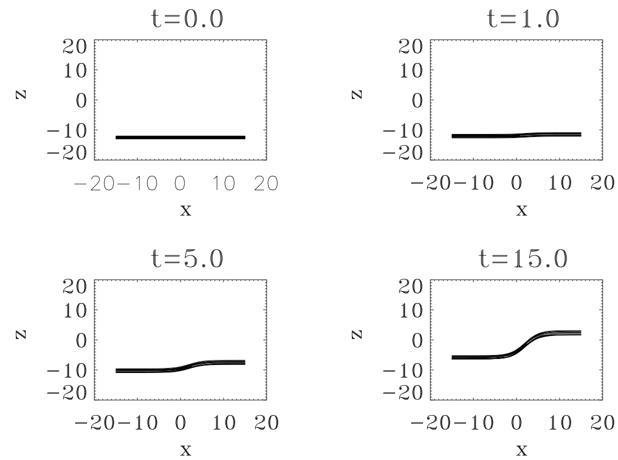

FIG. 1: Contourplots of the evolution of an initially plane single Alfvénic pulse in a plasma with an Alfvén speed varing in the x-direction. The unperturbed magnetic field is straight and has the z-component only. Here $A = 0.5$, $\lambda = 0.31$, $\beta = 2$.

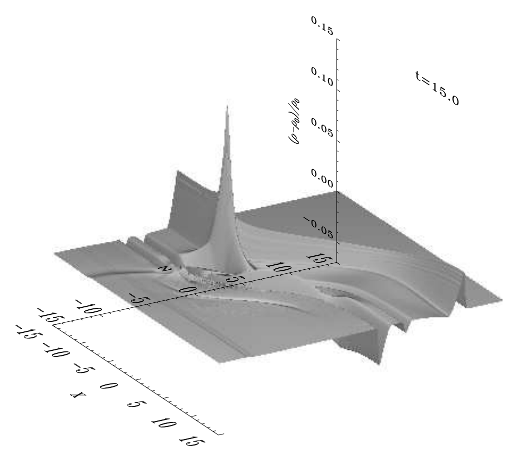

FIG. 2: A snapshot of the density perturbation at $t = 15$, generated by an Alfvénic pulse of the relative amplitude 0.5, in a plasma with $\beta = 2$ and inhomogeneity parameter $\lambda = 0.31$.

stage of this phenomenon could be performed with very high numerical resolution that became possible only recently. Indeed, it was found [11] that in the weakly nonlinear regime, phase-mixing leads to quick saturation of the interaction. In the regime, the saturation does not allow the fast wave amplitude to grow to more than a few tenth of a percent of the initial Alfvén wave amplitude. However, the tendency of the saturation level to grow with the Alfvén wave amplitude was noticed, which indicated that in the case of higher amplitudes, the generation of compressible waves can be significant. In the strongly nonlinear regime, in addition to phase mixing, coupling of MHD waves is also affected by the steepening of the Alfvén wave [5], and by the back reac-

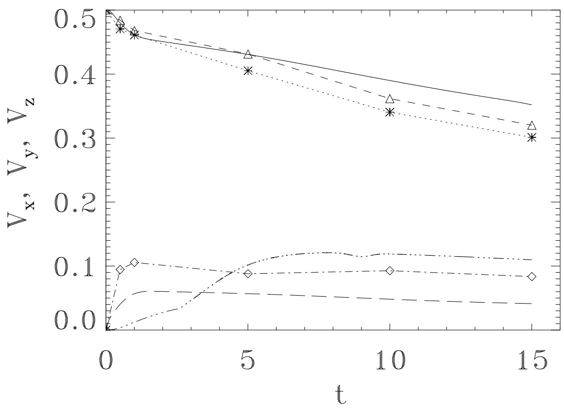

FIG. 3: Evolution of maximal values of the relative amplitude of the generated compressible perturbations, transverse $max(|V_x(x,z,t)|)$ and longitudinal $max(|V_z(x,z,t)|)$ and that of Alfvén wave $max(|V_y(x,z,t)|)$, in time. The solid curve presents decay of the initial Alfvén perturbation due to shock dissipation in the case of no plasma density inhomogeneity ($\lambda = 0$). The short dashed curve with triangles depicts $max(|V_y(0,z,t)|)$ when $\lambda = 0.31$, while dotted curve with astersics represents the same physical quantity on the edge of the similation box (away from the phase-mixing region) i.e. $max(|V_y(15,z,t)|)$. The dash-dotted curve with diamonds represents $max(|V_z(x,z,t)|)$, while the dash-triple-dotted curve shows $max(|V_x(x,z,t)|)$ both for the case of $\lambda = 0.31$. The long dashed curve represents $max(|V_z(x,z,t)|)$ for the case when $\lambda = 0$. Here $A = 0.5$ and $\beta = 2$.

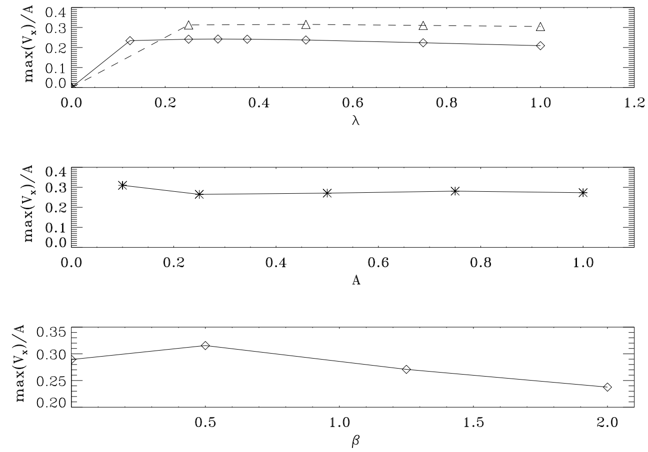

FIG. 4: **Top panel**: Maximal values of the relative amplitude of the generated compressible perturbations for different values of the inhomogeneity parameter λ, with $\beta = 2$ (solid curve) and $\beta = 0.5$ (dashed curve). The initial amplitude of the Alfvénic pulse is $A = 0.5$. **Mid panel**: Maximal values of the relative amplitude of the generated compressible perturbations for different values of initial amplitude of the Alfvénic pulse, with inhomogeneity parameter $\lambda = 0.5$ and $\beta = 1.25$. **Bottom panel**: Maximal values of the relative amplitude of the generated compressible perturbations for different values of β, with initial amplitude of the Alfvénic pulse $A = 0.5$ and inhomogeneity parameter $\lambda = 0.5$.

tion of the generated fast wave on the Alfvén wave. Indeed, as it has been shown by numerical simulations [7], the nonlinear interaction of magnetoacoustic and Alfvén waves causes energy transfer to smaller and smaller spatial scales. This phenomenon may lead to the formation of shock waves and, therefore, to increased heating of plasma by viscosity and resistivity. In modelling of non-resonant interaction of MHD waves on a plasma inhomogeneity, we consider a plasma configuration similar to the one investigated in [7] and [9], i.e. the plasma has one-dimensional inhomogeneities in the equilibrium density $\rho_0(x)$ and temperature $T_0(x)$, and is penetrated by a straight and homogeneous magnetic field directed along the z-axis. The unperturbed total pressure is taken to be constant everywhere. The density profile is a smooth interface,

$$\rho(x) = 3 - 2\tanh(\lambda x), \tag{4}$$

where λ is a parameter prescribing the steepness of the profile near $x = 0$. The density is normalized to $\rho_0(x = 0)$. The sharpest gradients are located near $x = 0$. Both the Alfvén speed $C_A(x)$ and the sound speed $C_s(x)$ are inhomogeneous in the x-direction, however the parameter β and the unperturbed total pressure are constant across the profile. Dynamics of the plasma is described by single-fluid MHD equations in Cartesian coordinates. The MHD equations are solved with *Lare2d* [12].

An initially plane linearly polarized Alfvénic pulse of Gaussian shape and highly nonlinear amplitude $V_y = 0.5C_A(0)$, propagates along the background magnetic field, in the z-direction. *There are no compressible perturbations present in the system at the beginning.* The pulse is subject to phase-mixing, which takes place in the vicinity of $x = 0$. Fig. 1 shows the development of the transverse gradients in the Alfvén pulse. Longitudinal and transverse gradients generate compressible perturbations (V_x, V_z, ρ, B_z and B_x), associated with slow and fast magnetoacoustic waves. Note, that initially plane Alfvén waves (perturbing V_y and B_y) in the *homogeneous* medium do *not* generate the perturbations of V_x, B_x and B_z. Thus, the generation of these perturbations in our simulations are caused by the effect of plasma inhomogeneity (see Fig. 3). It is clearly seen (Fig. 2) that the relative perturbations of the plasma density are *enhanced by a factor of about two* in the phase-mixing region (near $x = 0$). As in the weakly-nonlinear regime ([11]), initially the compressible disturbances grow secularly, according to Eq. (3), and then a saturated state is approached (Fig. 3). The saturation time is proportional to the efficiency of the generation. However, the maximal relative amplitude of compressible perturbations is *practically independent of the problem paramteres.* The parametric study (Fig. 4, top and mid panel) demonstrates that the efficiency of the non-resonant genera-

tion of compressible disturbances by phase-mixed Alfvén waves depends weakly upon the initial Alfvén wave amplitude and the steepness of the inhomogeneity, represented by the parameter λ (see Eq. (4)). The dependence on the plasma parameter β is more pronounced (see Fig. 4, bottom panel), and the highest amplitude of compressible disturbances is reached for $\beta \approx 0.5$. *For all investigated parameters, the observed saturation level was about 30% of the initial Alfvén wave amplitude.*

We conclude that in the presence of plasma inhomogeneity across the magnetic field, incompressible (Alfvén) and compressible (magnetoacoustic) MHD fluctuations are effectively coupled by non-resonant mechanisms. This coupling takes place even if the waves are linearly decoupled, e.g. the perturbations are plane in the ignorable direction perpendicular to both the magnetic field and the inhomogeneity gradient. These results demonstrate that the efficiency of the nonlinear interaction of the different type modes is strongly affected by the inhomogeneity of the medium. In particular, in the presence of a one-dimensional inhomogeneity, nonlinear generation of compressible disturbances by incompressible Alfvén waves is enhanced up to a factor of two (Fig. 3). The amplitude of the generated compressible disturbances experiences saturation at the level of about 30% of the initial amplitude of the source Alfvén wave. For example, it is 10-20% of the background density for the Alfvén wave amplitude of about 0.5. In particular, *this result is relevant to the explanation of the absence of a significant compressible component in the solar wind MHD turbulence*. Indeed, the density fluctuations of less than 30% of the background value, prescribed by the saturation, can hardly be detected in a high-β plasma because of a high level of the thermal noise in the *in situ* data. These results are especially relevant to the interpretation of observational data obtained by CLUSTER-II mission. This issue will be discussed in more detail elsewhere.

Finally, we would like to point out that the proposed mode coupling mechanism could find applications in other branches such as propagation of elastic waves in a inhomogeneous, fluid-saturated porous medium, as mathematically, the equations describing this phenomenon are similar to the MHD equations [13].

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